

that the reader knows Measure Theory or Lebesgue Integration. These things are briefly summarized in an appendix to Chapter Four.

The only serious mistake I found is in connection with the Principle of Uniform Boundedness and its application to the Lax Equivalence Theorem. It is assumed, (5.47) and also p. 155, that the semigroup  $E(t)$ , the solution operator in  $L_2$  to the homogeneous heat equation, is continuous with respect to  $t$  in the operator norm. This is false. However, for the Lax Equivalence Theorem it suffices that  $E(t)v$  is continuous in  $t$  for each fixed  $v$  in  $L_2$ , also known as “strong convergence”. The mistake is embarrassing since the proof of (5.47) is given as a problem, no. 5.29. The “solution” given to this problem ends by purporting to have shown a certain inequality. This inequality actually shows only “strong convergence”. However, the proof of even this weaker result is completely wrong, although the result is true.

I also found two problems that I regard as “marginally misleading”. Problem 8.27 might lead the unwary student to deduce a result which is true only in one space dimension. Interpreting the undefined and unlisted symbol  $\mathcal{C}_0^2(\bar{\Omega})$  in the most charitable sense, one looks for solutions of a “smooth” second order elliptic problem  $Lu = f \in \mathcal{C}(\bar{\Omega})$  with homogeneous Dirichlet conditions in  $\mathcal{C}^2(\Omega) \cap \mathcal{C}^0(\bar{\Omega})$  which are zero on  $\partial\Omega$  (classical solutions). In more than one space dimension the problem is correct only by virtue of its hypotheses always being false. (E.g., the problem  $\Delta u = f$  in  $\Omega$  a bounded domain in  $R^n$ ,  $n \geq 2$ ,  $u = 0$  on  $\partial\Omega$  nice, will *not* have a classical solution for every  $f$  in  $\mathcal{C}(\bar{\Omega})$ .) Problem 8.20 might lead the unsuspecting reader to believe that the Legendre series for a function in  $\mathcal{C}^2([-1, 1])$  converges only to order  $n^{-1/2}$  in the maximum norm; this is a full order  $n^{-1}$  off.

My impression is that in spite of minor flaws this is an excellent text for a stimulating one-year course in Functional Analysis with applications.

L. B. W.

**6[65–01].**—J. F. BOTHA & G. F. PINDER, *Fundamental Concepts in the Numerical Solution of Differential Equations*, Wiley, New York, 1983, xii + 202 pp., 24 cm. Price \$24.95.

This volume is intended for the novice practitioner who needs to read up quickly on basic practical numerical methods for (mainly) partial differential equations. It by and large bypasses theory, but some understanding and much practical advice is given. In the above respects it resembles von Rosenberg’s brief volume, [2]. von Rosenberg’s book treated finite difference methods, whereas the present exposition, written a decade and a half later, gives equal space to Galerkin Finite-Element methods, Collocation Finite-Element methods, and Boundary Element methods. It can be viewed as a pared-down version of Lapidus and Pinder [1]. I quote from the Preface: “This book is designed to provide an affordable reference on the methodology available for the solution of ordinary and partial differential equations in science and engineering.”

It is hard to judge whether the authors have succeeded in their goal. Certainly the treatment is brisk and to the point and always elucidated with examples showing the “how-to” of the method. A basic tenet is said to be the following: “The various

numerical methods are developed from one fundamental base—the theory of interpolation polynomials.” Such a foundation is rather proper in one-dimensional situations, cf. the elegant little book by Wendroff, [3]. In more than one dimension the claim is somewhat overstated. You will need much more than interpolation polynomial theory to conquer a given partial differential equation.

The brisk treatment means that the authors often give their own opinions in practical matters without supporting evidence. For example, on page 36 the following statement appears: “It is enough to say here that the application of Galerkin’s method to first order equations is never worth the effort.”

There are a few questionable statements in the book. The worst one appears in Section 1.4, pp. 5–6, on why and how boundary conditions must be specified. For example, for  $\Delta u = 0$  in a two-dimensional region, since there are 4 derivatives involved, formal integration gives 4 unknown functions! Happily, if the region is rectangular, there are 4 sides and so we can specify the correct number of conditions! Woe if you were to solve the problem in a triangle or a pentagon or a disc! For the heat equation it is only by convention that one specifies initial time conditions (in addition to spatial boundary conditions). Final time conditions would be equally appropriate, it appears!

For the interested reader, I will record that I also found statements that I quarrel with on the following pages: 30, 49, 52, 55, 77, 97, 100–107, 117. Happily, most of these statements can be taken as starting-points for constructive discussions. Most likely, the authors have evidence which they do not present in this brief volume.

The practical hints given seem mostly to pertain to problems which require only low-accuracy solutions, say 5–10 percent relative error in multidimensional situations. A scientist who desires to illustrate her/his theory by compelling numerical examples might well heed different advice.

Having thus said that the book contains statements that merit reflection, for novice and expert alike, I point out that it is indeed a brisk and to the point introduction to numerical methods in partial differential equations. Most major classes of methods are treated in some detail. The “how-to” is explained with detailed examples, and the authors share their wealth of knowledge in practical evaluation of the methods. This volume should serve at least as an introduction to its stated purpose, “...to supply the inexperienced scientist or engineer with the fundamental concepts required to achieve this objective (to obtain a relevant numerical solution efficiently and accurately)”.

L. B. W.

1. L. LAPIDUS & G. F. PINDER, *Numerical Solution of Partial Differential Equations in Science and Engineering*, Wiley, New York, 1982.

2. D. V. VON ROSENBERG, *Methods for the Numerical Solution of Partial Differential Equations*, Gerald L. Farrar and Associates, Inc., Tulsa, 1969.

3. B. WENDROFF, *First Principles of Numerical Analysis*, Addison-Wesley, Reading, Mass., 1969.

**7[65R20, 76B05].**—H. SCHIPPERS, *Multiple Grid Methods for Equations of the Second Kind with Applications in Fluid Mechanics*, Mathematical Centre Tracts 163, Mathematisch Centrum, Amsterdam, 1983. iii + 133 pp., 24 cm. Price \$6.00.

This Mathematical Centre Tract has been based on the author’s Ph.D. Thesis at Delft University of Technology (with Professor Wesseling). It provides a well-written